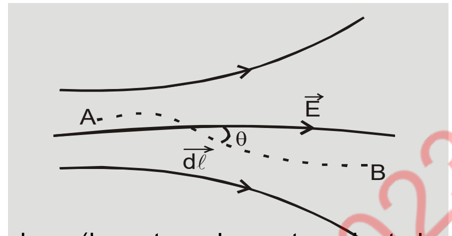
LINE INTEGRAL OF ELECTRIC FIELD:

The line integral of electric fields is defined as the integral

$$\int_A^B \vec{E}.\vec{dl}$$

The value of line integral depends only on the position of points A and B, and is independent of the path between A and B



$$\int_A^B \vec{E}.\vec{dl} = -\int_B^A \vec{E}.\vec{dl}$$

Line integral for a closed path is zero $\oint \vec{E}.\vec{dl} = 0$

5. ELECTRIC POTENTIAL:

Potential at any point A is equal to the amount of work done (by external agent against electric field) in bringing slowly a unit positive charge from infinity to that point.

$$V_A = \frac{W_{\infty A}}{q}$$

Unit of potential (V) = J/C or volt

Potential at a point is said to be one volt if the amount of work done in bringing one coulomb of positive charge from infinity to that point is one joule.

Since work and charge, both are scalars, the electric potential is a scalar quantity.

The dimensions of electric potential are

or $[V] = ML^2T^{-3}A^{-1}$

POTENTIAL DIFFERENCE:

Potential difference between two points f (final) and i (initial) is the amount of work done (by external agent) in moving slowly a unit positive charge from point i (initial) to f (final)

$$V_f - V_i = \frac{W_{i\,f}}{q}$$

If work done in carrying a unit positive charge from point 1 to point 2 is one joule then the potential difference $V_2 - V_1$ is said to be one volt.

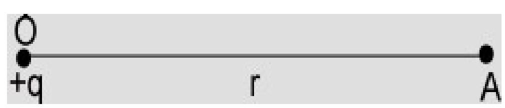
Potential difference may be positive or negative.

The work done against electrical forces in transporting a charge q from point i (potential V_i) to point f (potential V_f) is W = qV where

$$V = V_f - V_i$$

ELECTRIC POTENTIAL DUE TO POINT CHARGES:

As $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$ and $V = -\int_\infty^r \vec{E}.d\vec{r}$



$$V = -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\ \vec{r}.d\vec{r} = -\frac{1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}dr$$

Or $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$,

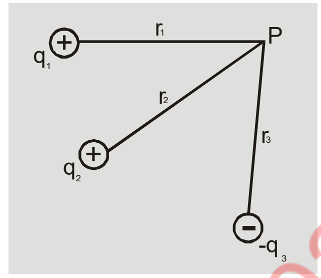
Where r is the distance of A from the point charge q.

The electric potential at $A(V_A)$ is positive if the point charge q is positive. V_A will be negative if the point charge q is negative.

MANY POINT CHARGES:

Potential is a scalar quantity and adds like scalers. Thus potential at point P (see fig.) due to charges $q_1, q_2, -q_3$ is equal to (algebraic) sum of potentials due to individual charges.

$$V = V_1 + V_2 + V_3 + ...$$



RELATION BETWEEN E AND ELECTRIC POTENTIAL V

$$\Delta V = V_f - V_i = -\vec{E}.\vec{\Delta l}$$

In one dimensions

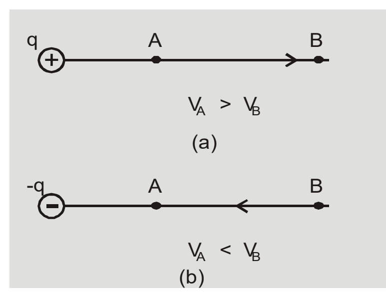
$$E = -\frac{dV}{dr} \qquad ......(1)$$

$$V = -\int E\, dr \qquad ......(2)$$

Electric field at any point is equal to negative of potential gradient at that point.The electric field always points from higher potential to lower potential (see fig.)

A positive charge always moves from higher potential to lower potential.

A negative charge always moves from lower potential to higher potential.



POTENTIAL DUE TO A CHARGED SPHERICAL SHELL:

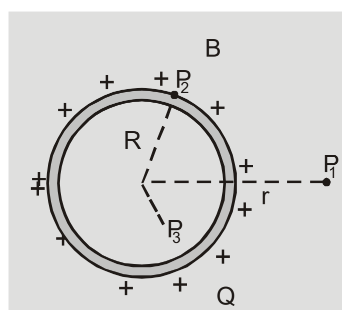
The charge resides on the shell surface.

The potential at P_1, outside point, is

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

The potential at P_2, surface point is

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$$



The potential at, P_3 inside point is

$$V = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

It is constant inside the shell (same at all points inside the shell, as on the surface)

Electric potential energy (U):

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge from an infinite distance.

U is a scalar quantity, dimension of [U] = ML^2T^{-2} , unit of [U] = joule

POTENTIAL ENERGY OF TWO POINT CHARGES:

The potential energy possessed by a system of two-point charges q_1 and q_2, separated by a distance r is the work required to bring them to this arrangement form infinity. This electrostatic potential energy is given by

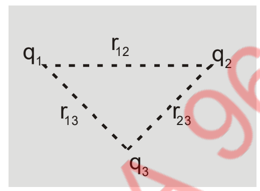
$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Note: While writing potential or potential energy charges must be multiplied with their respective signs.

FOR THREE CHARGES:

$U = U_{12} + U_{23} + U_{13}$

$$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$



DIELECTRIC STRENGTH:

The electric strength of air is about 3×10^6 V/m or 3000 V/mm. This means that if the electric field exceeds this value sparking will occur in air. This sets a limit on maximum charge that can be given to a conducting sphere in air.

The dielectric strength sets a limit of the maximum charge that can be placed on a conductor.

Electron volt

It is equal to the amount of energy gained by an electron when accelerated through a potential difference of one volt. It is unit of energy.

1 eV = 1.6×10^{-19} joule

When a charged particle moves under the influence of an electric field, then,

Kinetic energy gained = Potential energy lost

Energy density

In electric field, energy stored per unit volume is called energy density. It is equal to

$$u = \frac{1}{2}\varepsilon_0 E^2$$

SOLVED EXAMPLESL:

Example.13 A uniform electric field of magnitude E_0 and directed along positive x-axis exist in a certain region of space. If at x = 0, potential V is zero, then what is the potential at x = $+x_0$?

Solution. $E = -\frac{dV}{dr}$

$V - 0 = -E(x_0 - 0)$ or $V = -Ex_0$

(Note the negative sign. As one moves along the direction of electric field, the potential falls)

Example.14 Electric field intensity is given by the relation E = 100/x^2 where x is in meters. Find potential difference between the points x = 10 m and x = 20 m.

Solution.
$$V = -\int_{x_1}^{x_2} E dx = -\int_{10}^{20} 100x^{-2} dx = 100\left[x^{-1}\right]_{10}^{20}$$
$$= 100\left[\frac{1}{10} - \frac{1}{20}\right] = 5\,volt$$

Example.15 Three charges, –q, Q, q are placed at equal distances on a straight line. If the total potential energy of the system of the three charges is zero, then q : Q = ...

Solution. Let d be the equal distance. The total potential energy of the system is,
$$U = U_{12} + U_{23} + U_{31}$$
$$U = \frac{1}{4\pi \in_0}\left[\frac{q_1 q_2}{d} + \frac{q_2 q_3}{d} + \frac{q_3 q_1}{2d}\right]$$
$$= \frac{1}{4\pi \in_0}\left[-\frac{qQ}{d} + \frac{Qq}{d} - \frac{q^2}{2d}\right] \text{ or } U = \frac{1}{4\pi \in_0}\frac{q}{d}\left(-Q + Q - \frac{q}{2}\right)$$
Since U = 0
$$\therefore \quad q^2 = 0 \Rightarrow q = 0 \Rightarrow \frac{q}{Q} = 0$$

EXERCISE:

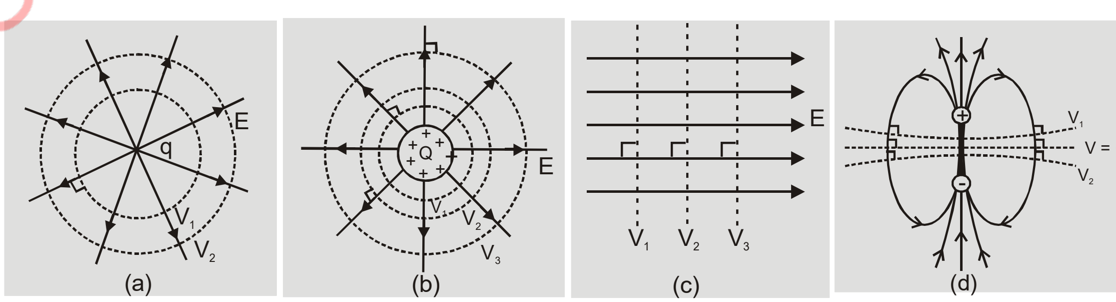
9. Thre concentric spherical shells have radii a, b and c (a < b < c) and have surface charge densities $\sigma, -\sigma$ and σ respectively. If V_A, V_B and V_C denote the potentials of the three shells, then, for c = a + b, we have
(1) $V_C = V_A \neq V_B$ (2) $V_C = V_B \neq V_A$ (3) $V_C \neq V_B \neq V_A$ (4) $V_C = V_B = V_A$

10. Two equal charges q are placed at a distance of 2a and a third charge –2q is placed at the midpoint. The potential energy of the system is
(1) $\frac{q^2}{3\pi\varepsilon_0 a}$ (2) $\frac{6q^2}{8\pi\varepsilon_0 a}$ (3) $-\frac{7q^2}{8\pi\varepsilon_0 a}$ (4) $\frac{9q^2}{8\pi\varepsilon_0 a}$

11. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9)$ volt. Value of electric field at x = 1 is
(1) –20 V/m (2) 6 V/m (3) 11 V/m (4) –23 V/m

6. EQUIPOTENTIAL SURFACE:

A surface on which the potential is constant is called an equipotential surface. (A curve on which the potential is constant is called equipotential curve)



Important Points Regarding Equipotential surface

(i) The electric field lines are perpendicular to equipotential surface (every where)

(ii) When a charge is moved on an equipotential surfaces, work done is zero

For a point charge q and spherical charge distributions, the equipotential surfaces are spherical fig. a & b dotted lines)

For a uniform field equipotential surface is plane (see fig. c) dotted lines) For a dipole, V = 0 surface is the equatorial plane.

Thus, in general equipotential surface can be of any shape.

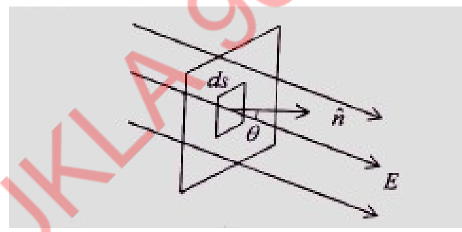
7. ELECTRIC FLUX:

The total number of electric lines of force passing perpendicular through the surface is called electric flux. It is given by the product of surface area and the component of intensity normal to area

$$d\phi = E\cos\theta . ds$$

$$d\phi = \vec{E} . \vec{ds}$$

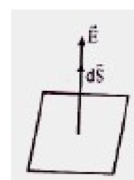
$$\phi = \oint \vec{E} . \vec{ds}$$



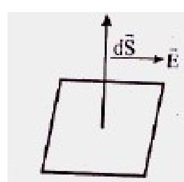
IMPORTANT POINTS REGARDING ELECTRIC FLUX:

(i) It is a scalar quantity and its unit is Nm^2/C or (volt × m)

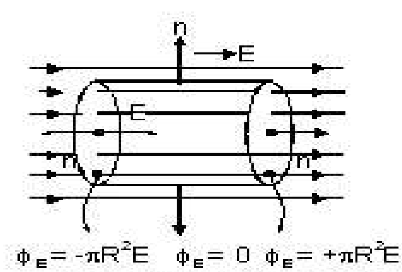
(ii) It will be maximum when $\cos\theta = \max. = 1,\ i.e.,\ \theta = 0^\circ$, i.e., electric field is normal to the surface with $(d\phi_E)_{\max} = \text{Eds}$



(iii) For $\theta = 90^\circ, \cos\theta = 0$ means ϕ_E is zero.



(iv) For a closed body outward flux is taken as positive while inward flux is taken as negative.



Cylinder in a uniform field

8. GAUSS'S LAW

This law gives a relation between the electric flux through any closed hypothetical surface (called a Gaussian surface) and the charge enclosed by the surface. It states. "The electric flux (ϕ_E) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the 'net' charge enclosed by the surface."

That is,

$$\varphi_E = \oint \vec{E}.\vec{ds} = \frac{\sum q}{\varepsilon_0}$$

Where $\sum q$ denotes the algebraic sum of all the charges enclosed by the surface.

If there are several charges $+q_1, +q_2, q_3, -q_4, -q_5......$ etc inside the Gaussian surface then

$$\sum q = q_1 + q_2 + q_3 - q_4 - q_5......$$

9. PROBLEMS RELATED TO ELECTRIC FLUX:

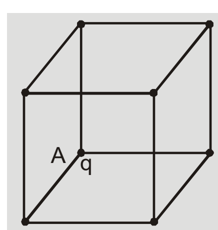
SOLVED EXAMPLES:

Example.16 If a point charge Q is located at the centre of a cube then find (i) flux through the total surface, (ii) flux through one surface.

Solution. (i) According to Gauss's law for closed surface $\phi = \frac{1}{\varepsilon_0} \times Q$, (ii) Since cube is a symmetrical figure thus by symmetry the flux through each surface is $\phi = Q/6 \in_0$

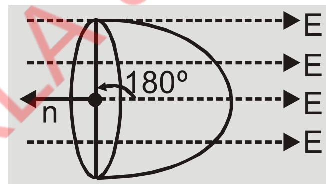
Example.17 A point charge q is placed at a corner of a cube with side L. Find flux through entire surface and flux through each face.

Solution. A corner of a cube can be supposed to be the centre of a big cube made up of 8 such cubes, therefore flux through it is $q/8 \in_0$.



The direction of E is parallel to the three faces that pass through this face, thus flux through these is zero.Flux through the other three faces $= \frac{1}{3}\left(\frac{q}{8 \in_0}\right) = \frac{q}{24 \in_0}$

Example.18 According to the figure, a hemi-spherical object is located in an electric field. Find the outward flux through its curved surface.



Solution. Total outward flux $\phi = \phi_{CS} = \phi_n$

Where ϕ_{CS} = flux through curved surface and

ϕ_n = flux through circular base

$\because$ No charge is associated with this surface

$$0 = \phi_{cs} + E\pi R^2 \cos 180^o$$

$$\phi_{cs} = E\pi R^2$$

Example.19 What is the value of electric flux in SI unit in Y-Z plane of area $2m^2$, if intensity of electric field is $\vec{E} = \left(5\hat{i} + 2\hat{j}\right)$ N/C

Solution. $\phi = \vec{E}.\vec{dA}$

$$= \left(5\hat{i} + 2\hat{j}\right)2\hat{i}$$

$$= 10\,\text{Nm}^2/\text{C}$$

EXERCISE:

12. A cube of side *l* is placed in a uniform field ***E***, where $\mathbf{E} = \mathbf{E}\hat{\mathbf{i}}$. The net electric flux through the cube is

(1) zero (2) l^2E (3) $4l^2$E (4) $6l^2$E

13. Eight dipoles of charges of magnitude e are placed inside a cube. The total electric flux coming out of the cube will be

(1) $\frac{8e}{\varepsilon_0}$ (2) $\frac{16e}{\varepsilon_0}$ (3) $\frac{e}{\varepsilon_0}$ (4) Zero

14. A point charge +q is placed at the centre of a cube of side L. The electric flux emerging from the cube is

(1) $\frac{q}{\varepsilon_0}$ (2) Zero (3) $\frac{6qL^2}{\varepsilon_0}$ (4) $\frac{q}{6L^2\varepsilon_0}$

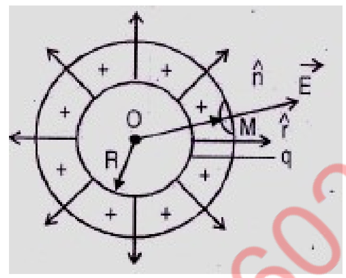
APPLICATIONS OF GAUSS'S LAW:

Gauss's law is useful when there is symmetry in the charge distribution, as in the case of uniformly charged sphere, long cylinders, and flat sheets. In such cases, it is possible to find a simple Gaussian surface over which the surface integral given by Gauss Law can be easily evaluated.

ELECTRIC FIELD TO A UNIFORMLY CHARGED SPHERICAL SHELL:

At a point outside the shell

Let us take a spherical shell of radius R having uniform charge q distributed on its surface. The electric lines of force will be directed radially outwards. Gaussian surface is a sphere of radius r. Thus, applying Gauss's Law. E at a point M, such that $OM = r > R$ is given by



$$\oint_S \vec{E}\,\vec{ds} = \oint_S \vec{E}.\hat{n}\,ds = \frac{q}{\varepsilon_0}$$

or electric field intensity $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

It is to be noted that the field outside a uniformly charged conducting sphere is the same as if whole of the charge was concentrated at the centre of the sphere.

At a point on the surface of the shell

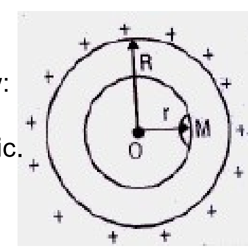
Here $r = R$

Therefore, $E = \dfrac{q}{4\pi\varepsilon_0 R^2}$

Or $E = \dfrac{\sigma}{\varepsilon_0}$, where $\sigma\left(= \dfrac{q}{4\pi R^2}\right)$ is the surface charge density of the shell.

At a point inside the shell

In this case, $r\,(< R)$ is the distance of point M from the centre O of the sphere. Imagine a Gaussian surface of the shape of a sphere of radius r.

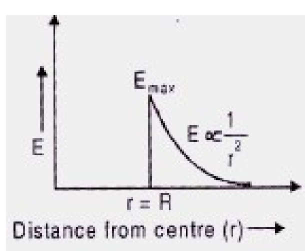


As no charge is enclosed by the Gaussian surface, therefore by Gauss's law:

$\oint \vec{E}.\vec{ds} = 0$ .Further since charge distribution and charged surface is symmetric.

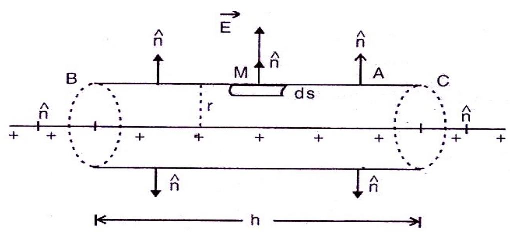
E should be constant on the imaginary surface. Hence, E = 0.

Hence, there is no electric field inside the charged spherical shell. All the three cases are represented graphically below. It shows the variation of electric field intensity E with distance from the centre of a uniformly charged spherical shell.



ELECTRIC FIELD DUE TO AN INFINITELY LONGSTRAIGHT CHARGED WIRE:

Take a small section of a infinite rod having charge density λ. We have to get an expression for electric field at any point M at a perpendicular distance r from the rod. Choose the Gaussian surface as a cylinder of height h, radius r and coaxial with the charged wire. The cylinder is closed at each end normal to the axis. The Gaussian surface consists of a curved surface A and ends of the cylinder B and C



The net electric flux

Flux through the entire Gaussian surface =

$$\phi = \oint_A \vec{E}.\vec{ds} + \oint_B \vec{E}.\vec{ds} + \oint_C \vec{E}.\vec{ds} + ... \quad ...(i)$$

Since at the ends of cylinder, angle between electric field intensity $\vec{E}$ and $\hat{n}$ is 90°.

$$\oint_B \vec{E}.\vec{ds} = \oint_C \vec{E}.\vec{ds} = 0 \quad ...(ii)$$

For the curved surface, $\vec{E}$ is constant everywhere. Also and the unit vector normal to curved surface are in the same direction such that $\theta = 0^o$.

Thus, the electric flux through the curved surface of cylinder is given by,

$$\phi = E(2\pi rh) \quad ...(iii) \text{ (Using (i) and (ii))}$$

Charge enclosed in the cylinder = liner charge density × Length Or

$$q = \lambda h \quad ...(iv)$$

By Gauss's law,

$$\phi = \frac{q}{\varepsilon_0} \quad ...(v)$$

From the equation (iii), (iv) and (v)

$$E(2\pi rh) = \frac{\lambda h}{\varepsilon_0}$$

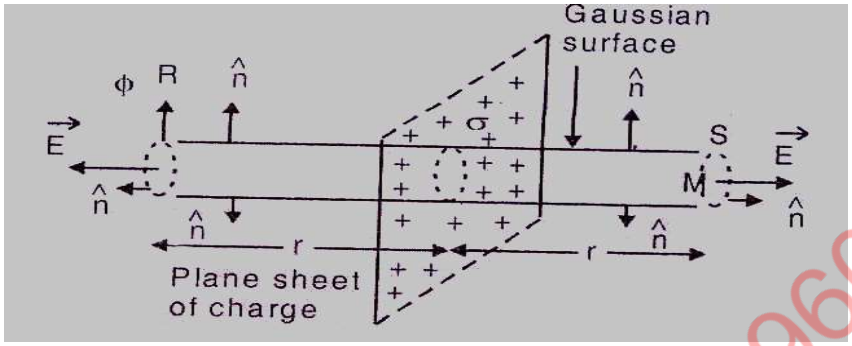
Or (Electric field intensity) $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

Clearly, $E \propto \dfrac{1}{r}$

Here, we see E is inversely proportional to the distance r from the line. And the direction of E is radially outward if the line of charge is uniform and positive and inward if it is uniform and negative.

INFINITE PLANE SHEET:

Consider a thin, infinite plane sheet of charge with surface charge density σ. We have to evaluate electric field intensity at point M, at a distance r from the sheet. Take a cylinder having cross-sectional area A and length 2r whose walls are perpendicular to the sheet of charge.



Assuming the cylinder as the Gaussian surface, by symmetry, the electric field on either sides of the sheet should be normal to the plane of sheet, having same magnitude at all points equidistant from the sheet. At the two cylindrical edges, R and S, $\vec{E}$ and $\hat{n}$ are parallel to each other as shown in the figure.

Now, electric flux over these edges $= 2\vec{E}.\hat{n}\,ds = 2E\ ds$

The components of electric field E normal to the walls are zero as no lines of force cross the sidewalls of the cylinder.

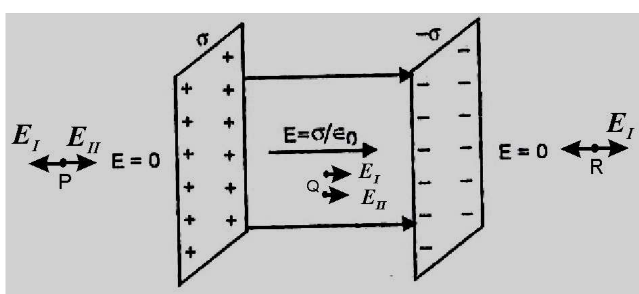
Therefore total electric flux over the entire surface of the cylinder $= 2\ E\ ds.$

Also, the total charge enclosed by the cylinder $= \sigma\ ds$

$2\,E\ ds = \dfrac{q}{\varepsilon_0} = \dfrac{\sigma ds}{\varepsilon_0}$ or electric field intensity $E = \dfrac{\sigma}{2\varepsilon_0}$ ... (i)

ELECTRIC FIELD INTENSITY DUE TO TWO THIN INFINITE PARALLEL SHEETS OF CHARGE:

$$E_P = \vec{E}_I + \vec{E}_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$



$$E_Q = \vec{E}_I + \vec{E}_{II} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$E_R = \vec{E}_I + \vec{E}_{II} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

Thus, Electric field is uniform between two plates carrying equal and opposite charge and is directed from positive to negative plate. Outside the plate electric field is zero.

PROBLEM RELATED TO GAUSS'S LAW:

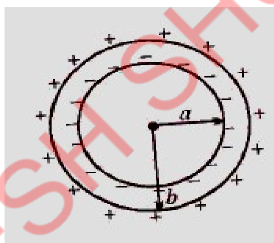
SOLVED EXAMPLES:

Example.20 In the figure shown below, a point q is placed at the center of neutral spherical conduction shell. What is the induced surface charge density at the inner and outer surface of the conducting shell?

Solution. Charge q will attract negative charge of the inner surface of the conductor. Charge will continue to move until the net field in the conductor due to q and induced charge is zero. Since the field of uniform spherical surface charge is the same as if it were concentrated at center, perfect cancellation occurs when exactly –q charge has been induced at the surface a. Therefore, the induced surface charge is $\sigma_a = -\frac{q}{4\pi a^2}$.

Since the conductor as a whole is neutral, there is a leftover charge +q on the conductor, which experiencing no force from q and σ_a (since their fields precisely cancel), distributes itself uniformly over outer surface, with a surface charge density

$$\sigma_b = \frac{q}{4\pi b^2}$$



Example.21 The electric field in a region is radially outward with magnitude E = Ar. Find the charge contained in a sphere of radius a centred at the origin. Take A = $100\ \text{V/m}^2$ and $a = 20.0\ cm$.

Solution. The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E\, ds \cos\theta = Aa\left(4\pi a^2\right)$$

The charge contained in the sphere is, from Gauss's law,

$$Q_{inside} = \varepsilon_0 \Phi = 4\pi\varepsilon_0 A a^3$$

$$= \left(\frac{1}{9\times10^9}\frac{C^2}{N-m^2}\right)\left(100\frac{V}{m^2}\right)(0.20\,m)^3$$

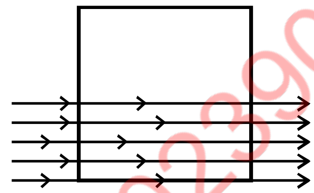
$$= 8.89\times10^{-11}\,C$$

EXERCISE:

15. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2 the electric charge inside the surface will be

(1) $(\phi_1 + \phi_2)\varepsilon_0$ (2) $(\phi_1 - \phi_2)\varepsilon_0$ (3) $(\phi_1 + \phi_2)/\varepsilon_0$ (4) $(\phi_1 - \phi_2)/\varepsilon_0$

16. A square surface of side L meters is in the plane of the paper. A uniform electric field $\vec{E}\,(\text{volt/m})$, also in the plane of the paper, is limited only to the lower half of the square surface, (see figure). The electric flux is SI units associated with the surface is



(1) zero (2) EL^2

(3) $EL^2/(2\varepsilon_0)$ (4) $EL^2/2$

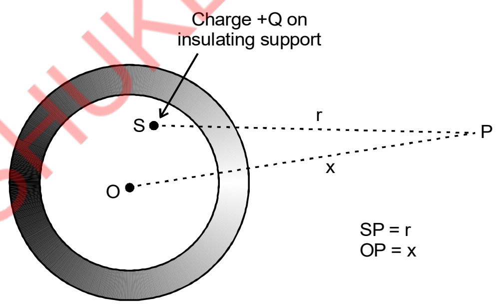
17. The adjacent diagram shows a charge +Q held on an insulating support S and enclosed by a hollow spherical conductor. O represents the centre of the spherical conductor and P is a point such that OP = x and SP = r. The electric field at point P will be

(1) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$

(2) $\dfrac{Q}{4\pi\varepsilon_0 r^2}$

(3) 0

(4) None of the above



CAPACITOR

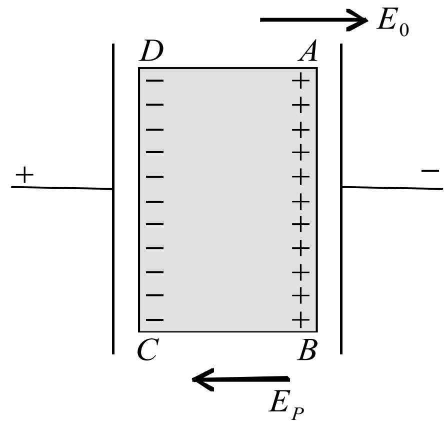
Conductor: - A substance which can be used to carry or conduct electric charge from one place to the other is called conductor. In a metallic conductor, there are a large number of free electrons which act as carriers of charge. Infact, one or more electrons per atom are free to roam about in the body of the metal, though they cannot leave the metal at normal condition. The residual atoms in the metal are positively charged. They constitue the bound charges in the conductor. In liquid conductors, it is the ions that acts as the charge carriers.

Insulator: - The insulators are the materials which cannot conduct electricity. Infact, in an insulator, each electron is attached or bound to a particular atom and is not free to move in the body of insulator. Insulators are also called dielectrics. Dielectrics are of two types (i) Non polar dielectrics (ii) Polar dielectrics.

BASIC PROPERTIES OF CONDUCTORS IN ELECTROSTATICS:

1. **Electric field is zero inside a conductor**

Net field inside the conductor, $E = E_0 - E_P = 0$



2. **Excess charges always lie on the surface**
   This follows from Gauss law. If we imagine any closed Gaussian surface which is just inside the surface, the electric flux through the surface is zero as there is no electric field inside the conductor, hence the net charge enclosed must be zero. Therefore any excess charge must reside on the surface.
3. **Potential is constant inside the conductor**

   If a and b are any two points within a given conductor, $V_b - V_a = \int_a^b \vec{E}.\vec{dl} = 0$ (E = 0), and hence

   $V(a) = V(b)$
4. **Electric field is perpendicular to the surface, just outside the conductor.**
   If it is not so, charge will immediately flow around the surface until it cancels the tangential component. (charge can not flow perpendicular to the surface since it is confined to the conductor). If σ is the surface

   charge density at a point on the surface Gauss law can be used to show that $E_\perp = \frac{\sigma}{\varepsilon_0}$
5. **Surface density of charge is different at different points.**

<u>Capacitance</u>: - Electrical capacitance of a conductor is a measure of ability of the conductor to store charge on it. If *q* is the charge on the conductor and *V* is the potential of the conductor then

$q \propto V \qquad \Rightarrow q = CV$

Where *C* is the constant and is called capacity or capacitance of the conductor. The value *C* depends on the shape and size of the conductor and also on the nature of medium in which the conductor is located. *C* does not depend upon the nature of material of the conductor.

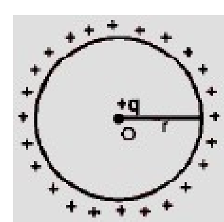
$Q = CV$ Let $V = 1$ then $Q = C$

So capacitance of a conductor is the amount of charge required to raise its potential by unity. Its unit is farad.

CAPACITY OF AN ISOLATED SPHERICAL CONDUCTOR:

When a charge Q is given to a spherical conductor of radius R, then

The potential of the surface of the spherical conductor will be $V = \frac{Q}{4\pi \epsilon_0 R}$



Electrical capacitance of spherical conductor in M.K.S. system will be

$$C = \frac{Q}{V} = \frac{Q}{Q/4\pi \epsilon_0 R} = 4\pi \epsilon_0 R$$

The electrical capacitance of a spherical conductor is directly proportional to its radiusi.e., C $\propto$ R. The electrical capacitance of a spherical conductor does not depend on the charge given to a conductor.

CAPACITOR (Condenser):

It is a useful device which stores charge and energy. It consists of two conductors insulated from each other. We place +Q charge on one conductor and –Q on the other. Since potential is constant over a conductor, one can speak unambiguously of the potential difference between them. Let V_+ and V_- be the potential of the conductors carrying positive and negative charge respectively. Then,

$$V = V_+ - V_- = \int_{(-)}^{(+)} \vec{E}.\vec{dl}$$

(Two insulated conductors carrying equal and opposite charges form a capacitor)

Now, since electric field is proportional to Q, so also is V. (If we double charge Q in the capacitor, electric field and potential also get doubled). Therefore, the ratio $\frac{Q}{V}$ is constant and is called the capacitance of the arrangement.

$$C = \frac{Q}{V}$$

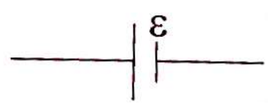
Capacitance C is a geometrical quantity, determined solely by the sizes, shapes and separation of the two conductors. In SI units, C is measured in farads (F). 1 Farad is one coulomb per volt. This is actually a very large unit. Commonly used unit is micro-farad where $1\mu F = 10^{-6} F$.

CHARGING OF A CAPACITOR BY A BATTERY:

To put equal and opposite charge on the two conductors of a capacitor, they may be connected to the terminals of a battery which transfers charge from one conductor to the other.

Properties of a battery:

1. A battery has two terminals and is represented by a symbol as shown in figure.



   The potential difference V between the terminals of the battery is constant. The terminal with higher potential is called the positive terminal and is represented by the longer line.

2. The potential difference V is equal to the electromotive force (emf) ε of the battery.

3. When a charge Q passes through the battery of emf from negative terminal to positive terminal, an amount Q of work is done by the battery.

PARALLEL-PLATE CAPACITOR:

It consists of two parallel plates say A and B.

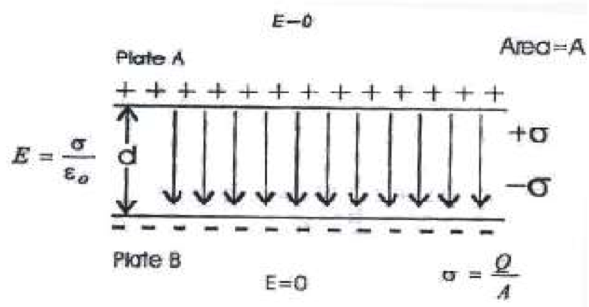
If we put Q charge on the top and –Q on the bottom, then it will spread out uniformly over the two surfaces provided the area is comparatively large and separation small. The surface charge density on the top plate is Q/A and -Q/A on the lower plate.

Then the electric field between the plate is uniform and is given as $E = \frac{\sigma}{\varepsilon_0}$

Since $E = \frac{V}{d}$

$\therefore \quad V = E.d = \frac{\sigma}{\varepsilon_0}.d$, where V is the

P.D. between the plate



$$C = \frac{Q}{V} = \frac{\sigma.A}{\frac{\sigma}{\varepsilon_0}.d} = \frac{\varepsilon_0 A}{d}$$

ε_0 = permittivity constant in free space

$$= 8.854 \times 10^{-12} \ F.m^{-1}$$

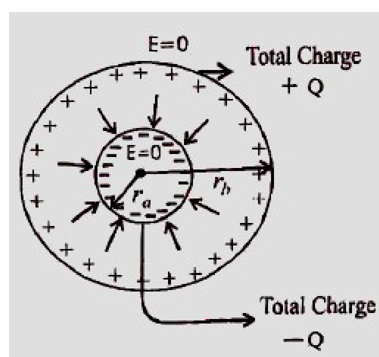
$$\therefore \quad C = \frac{\varepsilon_0 A}{d}$$

SPHERICAL CAPACITOR

A spherical capacitor consist of two concentric conducting shells A and B of radii r_a and r_b respectively.

To find capacitance, we place charge –Q on the inner conductor and +Q on the outer conductor and find the potential difference. The electric field between the shells is the same as that due to point charge Q at the origin. The electric field in the region $r < r_a$ and $r > r_b$ can be shown to be zero by using gauss law.

Hence, the electrostatic field between shells A and B is



$$E = \frac{-Q}{4\pi\varepsilon_0}.\frac{1}{r^2} \qquad \text{when } r_0 < r < r_b$$

$$E = 0 \qquad \text{when } r < r_a \text{ and } \qquad \text{when } r > r_b$$

The potential difference between the two spheres,

$$V = V_{(+)} - V_{(-)}$$

$$= \int_{r_a}^{r_b} E(r)\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

The capacitance of spherical capacitor is thus

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0.r_a.r_b}{r_b - r_a} = 4\pi\varepsilon_0 r_a\left[\frac{1}{1 - r_a / r_b}\right]$$

As the radius of the outer shell approaches infinity, the capacitance becomes $4\pi\varepsilon_0 r_a$. We can think of this result with the outer shell at infinity as the capacitance of a single isolated sphere. Therefore, the capacitance of an isolated sphere of radius R is $C = 4\pi\varepsilon_0 R$

APPLICATION OF CAPACITOR:

(i) Tuning in Radio circuits

(ii) Elimination of sparking in switches.

(iii) Timing circuit.

(iv) Nuclear fusion to store charges

ELECTROSTATIC ENERGY STORED IN CAPACITOR:

A charged capacitor stores an electric potential energy in it, which is equal to the work required to charge it. This energy can be recovered if the capacitor is allowed to discharge. If the charging is done by a battery, electrical energy is stored at the expense of chemical energy of battery.

Suppose at time t, a charge q is present on the capacitor and V is the potential of the capacitor. If dq amount of charge is brought against the forces of the field due to the charge already present on the capacitor, the additional work needed will be

$$dW = (dq)V = \left(\frac{q}{C}\right).dq \qquad (as\, V = q/C)$$

$\therefore$ Total work to charge a capacitor to a charge q_0,

$$W = \int dW = \int_0^{q_0}\left(\frac{q}{c}\right).dq = \frac{q_0^2}{2C}$$

Energy stored by a charged capacitor,

$$U = W = \frac{q_0^2}{2C} = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

Thus, if a capacitor is given a charge , the potential energy stored in it is,

$$u = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

ELECTROSTATIC FIELD ENERGY:

Energy density U is defined as the total energy stored per unit volume of the condenser.

$$U = \frac{\text{total energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} \qquad \left[\because C = \frac{A\varepsilon_0}{d}\right] \& \; [V = Ed]$$

$$\therefore U = \frac{\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\times(Ed)^2}{Ad}$$

$$\therefore U = \frac{1}{2}\varepsilon_0 E^2$$

SERIES COMBINATION OF CAPACITORS:

When capacitors are connected in series, the magnitude of charge Q on each capacitor is same. The potential difference across C_1 and C_2 is different i.e., V_1 and V_2.

$$Q = C_1 V_1 = C_2 V_2$$

The total potential difference across combination is:

$$V = V_1 + V_2$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

The ratio Q/V is called as the equivalent capacitance C between point a and b

The equivalent capacitance C is given by: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

The potential difference across C_1 and C_2 is V_1 and V_2 respectively, given as follows:

$$V_1 = \frac{C_2 V}{C_1 + C_2} \;\&\; V_2 = \frac{C_1}{C_1 + C_2} V$$

In case of more than two capacitors, the relation is:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \ldots\ldots$$

$$\Rightarrow \frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

PARALLEL COMBINATIONS OF CAPACITORS:

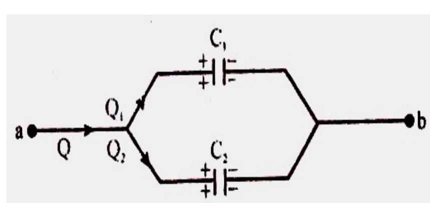
When capacitors are connected in parallel, the potential difference V across each is same and eh charge on C_1, C_2 is different i.e., Q_1 and Q_2.

The total charge is Q is given as:

$$Q = Q_1 + Q_2$$

$$Q = C_1 V + C_2 V$$

$$\frac{Q}{V} = C_1 + C_2$$



Equivalent capacitance between a and b is:

$$C = C_1 + C_2$$

The charges on capacitors is given as:

$$Q_1 = \frac{C_1}{C_1 + C_2} Q$$

$$Q_2 = \frac{C_2}{C_1 + C_2} Q$$

In case of more than two capacitors,

$$C = C_1 + C_2 + C_3 + C_4 + C_5 + ...$$

DIELECTRICS:

A non-conducting material such as glass or wood is called a dielectric. Though the electrons in such materials remain bound within their molecules and thereby preserving the overall neutrality of each molecule, they are affected by external electric field because positive nucleus (protons) and negative charges (electrons) tend to shift in opposite directions.There are two types of dielectrics:

1. **Non-polar dielectrics:**

   In this type of materials, the center of mass of all positive charges in a molecule coincides with the center of mass of all negative charges (electrons) in the absence of external electric field. Hence, they are not only electrically neutral but also have zero dipole moments.

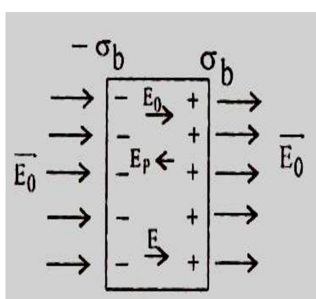
   Now, when external field is switched on, the two centres of charge get slightly separated and each molecule becomes a dipole, having a small dipole moment. This happens because protons move in the direction of electric field while electrons experience force in the opposite direction. Hence in the presence of electric field, the dielectric gets polarized.

2. **Polar dielectrics:**

   In polar dielectric, such as water, the center of mass of the protons in a molecule does not coincide with the center of mass of electrons even in the absence of electric fields. This happens because of asymmetric shape of the molecules. Thus each molecule behaves as dipole having permanent dipole moment.

   In the presence of external electric field, these dipoles tend to align themselves with the electric field.

DIELECTRIC CONSTANT K:

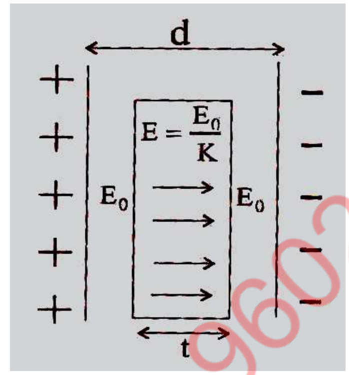


Suppose a dielectric slab is placed in a uniform electric field $\vec{E_0}$

The electric field will polarize the slab. The positive charges of all the molecules will be shifted towards right and negative charges will be shifted towards left. The net effect of the polarization is to produce a positive surface charge on the right face and a negative surface charge on the left surface. This surface charge sets up an electric field $\vec{E_P}$ in the opposite direction with is less than $\vec{E_0}$.

The net field inside the slab become $\vec{E} = \vec{E_0} + \vec{E_P}$.

The ratio $K = \frac{E_0}{E}$ is called the dielectric strength of the material. The induced charge appearing on the two surfaces is not free to move and is called bound charge.



PARALLEL PLATE CAPACITOR WITH DIELECTRIC:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

1. If the dielectric is completed filled, $t = d$, $C = \frac{K\varepsilon_0 A}{d}$

2. If Metallic slab($K = \infty$)of thickness t is introduced between the plates of capacitor $C = \frac{\varepsilon_0 A}{d - t}$

PROBLEM BASED ON CAPACITOR:

SOLVED EXAMPLES:

Example.22 The capacity of a parallel plate capacitor in air is and on immersing it into oil it becomes. The dielectric constant of oil is

[1] 0.45 [2] 0.55 [3] 1.10 [4] 2.20

Solution. K = $\frac{C}{C_0}$

K = $\frac{110}{50} = \frac{11}{5}$ = 2.20

Hence the correct answer will be (4)

Example.23 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

[1] 86 Pf [2] 96 pF [3] 90 pF [4] 80 pF

Solution. For parallel plate capacitor

C = $\frac{K \epsilon_0 A}{d}$, with K = 1 for air

If distance between plates is reduced to half and K = 6, the new capacitance is

$$C' = 6 \times \frac{\epsilon_0 A}{\left(\frac{d}{2}\right)} = 12. \frac{\epsilon_0 A}{d}$$

= 12 × 8 pF = 96 Pf

Example.24 A parallel plate condenser is charged to a certain potential and then disconnected.The separation of the plates is now increased by 2.4 mm and a plate of thickness 3 mm is inserted into it keeping its potential constant. The dielectric constant of the medium will be

[1] 5 [2] 4 [3] 3 [4] 2

Solution. As charge and potential of the condenser both are constant in two cases, hence its capacity must also remain constant

$C_0 = C$

or $\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left[1 - \frac{1}{K}\right]}$

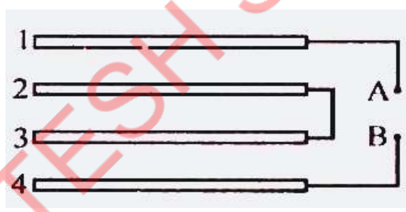
or $d = d' - t\left[1 - \frac{1}{K}\right]$ or $(d' - d) = t\left[1 - \frac{1}{K}\right]$

or $2.4 \times 10^{-3} = 3 \times 10^{-3}\left[1 - \frac{1}{K}\right]$ or $1 - \frac{1}{K} = 0.8$ or $\frac{1}{K} = 0.2$

$\therefore \quad K = 5$

Hence the correct answer will be (1)

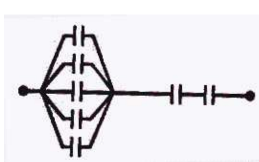
Example.25 Four identical plates each of area A are kept in air as shown in figure. Separation between each plate is d. Plates 2 and 3 are connected by a conducting wire. Find equivalent capacitance of system between A & B.

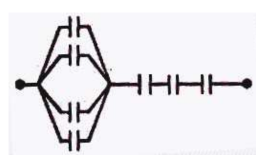


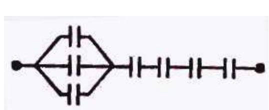
[1] $\frac{\varepsilon_0 A}{d}$ [2] $\frac{2\varepsilon_0 A}{d}$ [3] $\frac{\varepsilon_0 A}{2d}$ [4] $\frac{3\varepsilon_0 A}{d}$

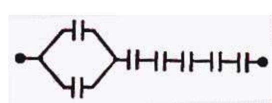
Solution. (3)

Example.26 Seven capacitors each of capacitance $2\mu F$ are to be connected in a configuration to obtain an effective capacitance of $(10/11)\mu F$. Which of the combination (s), shown in figure below, will achieve the desired result?

[1] 

[2] 

[3] 

[4] 

Solution. (1)

EXERCISE:

18. The potential energy of a charged parallel plate capacitor is U_0. If a slab of dielectric constant k is inserted between the plates, then the new potential energy will be

(1) $\frac{U_0}{k}$ (2) U_0k^2 (3) $\frac{U_0}{k^2}$ (4) U_0^2

19. A capacitor of capacitance value $1\mu F$ is charged to 30 V and the battery is then disconnected. If it is connected across a $2\mu F$ capacitor, the energy lost by the system is

(1) $300\mu J$ (2) $450\mu J$ (3) $225\mu J$ (4) $150\mu J$

(5) $100\mu J$

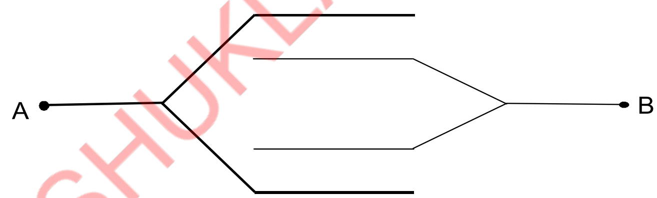
20. Force of attraction between the plates of a parallel plate capacitor is

(1) $\frac{q^2}{2\varepsilon_0 AK}$ (2) $\frac{q^2}{\varepsilon_0 AK}$ (3) $\frac{q}{2\varepsilon_0 A}$ (4) $\frac{q^2}{2\varepsilon_0 A^2 K}$

21. Four plates of equal area A are separated by equal distances d and arranged as shown in the figure. The equivalent capacity is

(1) $\frac{2\varepsilon_0 A}{d}$ (2) $\frac{3\varepsilon_0 A}{d}$

(3) $\frac{4\varepsilon_0 A}{d}$ (4) $\frac{\varepsilon_0 A}{d}$



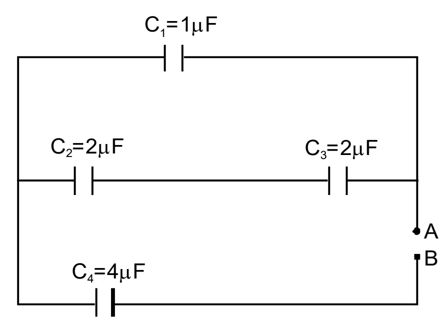
22. For capacitors are connected in a circuit as shown in the following figure. Calculate the effective capacitance between the points A and B

(1) $\frac{4}{3}\mu F$

(2) $\frac{24}{5}\mu F$

(3) $9\mu F$

(4) $5\mu F$



23. Six capacitors each of capacitance of $2\mu F$ are connected as shown in figure. The effective capacitnace between A and B is

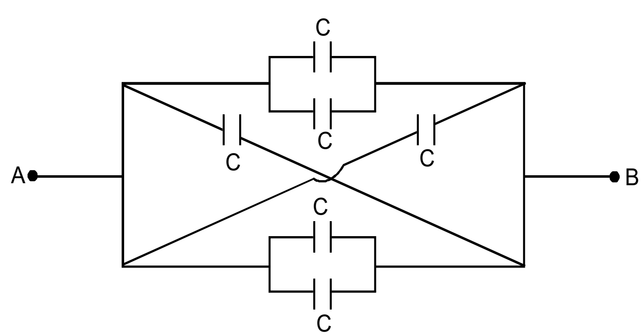
(1) $12\mu F$

(2) $8/3\mu F$

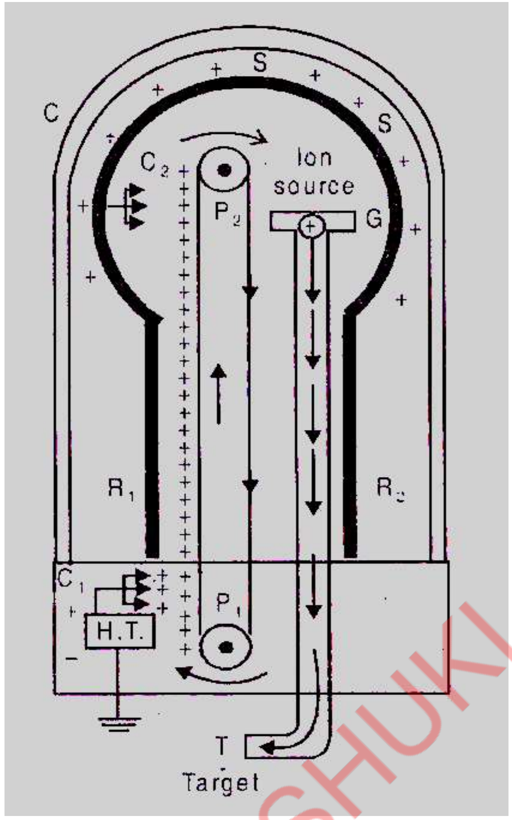
(3) $3\mu F$

(4) $6\mu F$

(5) $2/3\mu F$



VANDER GRAAFF GENERATOR:



It is a man made high voltage generator. It's a device which produces a high potential difference of about 10 million volts and it can be used as an accelerator of charged particles.

PRINCIPLE:

1. **The action of sharp points (Corona discharge):** The electric discharge takes place in air or gases readily at pointed conductors.
2. The property that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

CONSTRUCTION:

- S is the hollow conducting sphere supported by insulating pillars R, & R_2.
- P_1 and P_2 are pulleys wounded by an insulating narrow belt, which runs continuously with the help of a motor. P_2 is at the centre of S and P_1 is at ground level.
- C_1 - Spray comb at the lower end of the belt, which sprays charge on the belt.
- C_2 - Collecting comb at P_2 that collects charges and passes it to S.
- C - Steel chamber that encloses the generator and is filled with nitrogen and methane at high pressure to prevent leakage of charge.

WORKING:

The spray comb is maintained at a positive potential $\left(\approx 10^4 \text{ V}\right)$ with respect to the earth by high-tension source H.T. As a result of discharging action of sharp points, a positively charged electric wind is set up, which sprays positive charge on the belt (i.e. corona discharge). As the belt moves, and reaches the sphere, a negative charge is induced on the sharp ends of collecting comb C_2 and an equal positive charge is induced on the farther end of C_2. This positive charge shifts immediately to the outer surface of conducting sphere 'S'. Owing to discharging action of sharp points of comb C_2, a negatively charged electric wind is set up. This neutralizes the positive charge on the belt. The uncharged belt returns down, collects the positive charge from C_1, which in turn is collected by C_2, This process is repeated and thus the positive charge on metallic sphere 'S' goes on accumulating.

The capacity of spherical shell S is expressed as

$C = 4\pi\varepsilon_0 R$ .....(i) ; where R is radius of the spherical shell.

Now, potential $V = \dfrac{q}{C}$

Using equation (i)

$$V = \frac{q}{4\pi\varepsilon_0 R} \quad .....(ii)$$

It can be seen from equation (ii) that the potential V of the spherical shell goes on increasing with increase in charge q.

The breakdown field of air is nearly $3 \times 10^6 \text{ Vm}^{-1}$. The moment, the potential of conducting shell exceeds this value, air around shell S gets ionized and leakage of charge starts. To prevent the leakage of charge from the sphere, the generator assembly is enclosed inside a steel chamber filled with nitrogen or methane at high pressure.

If q' is the charge on the ion to be accelerated and V be the potential difference developed across the ends of the discharge tube, then energy acquired by the ions $= Vq'$. The ions hit the target with this energy and carry out the purpose.